ARAB REPUBLIC OF EGYPT
Ministry of Education
General Secondary Education Certificate Examination, ٢٠١٢
[Second Stage – First Session]

Algebra and Solid Geometry 【 Mathematics (٢) 】 Time: ٢ Hours

الجبر والهندسة الفراغية [رياضيات (٢)] باللغة الإنجليزية

تنبيه مهم : ١ـ يسلم الطالب ورقة امتحانية باللغة العربية مع الورقة المترجمة . 【 الأسئلة في صفحتين 】
٢ـ الإجابات المكررة عن أسئلة الاختيار من متعدد والصواب والخطأ لن تقدر ويتم تقدير الإجابة الأولى فقط .

Calculators are allowed: الدرجة الفعلية = مجموع الدرجات ÷ ٢

I – Algebra

Note that: $1, \omega, \omega^2$ are the cubic roots of unity, and: $i^2 = -1$

Answer TWO ONLY of the Following Questions:

١ – (٨ Marks)

(a) If $^{n+2}P_r = 2\ ^{n+2}C_r$, $\dfrac{^{n}C_{r+1}}{^{n}C_r} = \dfrac{5}{3}$,

find the value of: $^{2n}C_{n-r} + {}^{n+3}P_{r-1}$

(b) Find the numerical value of the expression:

$$\frac{1}{4\omega^3+2\omega^2+3\omega} + \frac{1}{5\omega^6+3\omega^4+4\omega^2}$$

٢ – (٨ Marks)

(a) If $Z = (1+\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})^6$, put the number Z in the trigonometry form, and then find the cubic roots of Z in the exponential form.

(b) Use the properties of determinants to find the values of x satisfying:

$$\begin{vmatrix} ٢x & ١ & ١ \\ ١ & ٢x & ١ \\ ١ & ١ & ٢x \end{vmatrix} = \text{zero}$$

٣ – (٨ Marks)

(a) In the expansion of $(x^2 + \frac{1}{2x})^{3n}$ according to descending powers of x

i) prove that the order of the term free of x is (٢ n + ١)

ii) Find the ratio between the value of the term free of x and the middle term at n = ٤ , x = ١

(b) Using Cramer's method, solve the following equations :

$$\begin{vmatrix} ٢ & y \\ ١ & x \end{vmatrix} = ١ , \quad \begin{vmatrix} ٣ & z \\ ٢ & y \end{vmatrix} = ١ , \quad \begin{vmatrix} ٣ & x \\ ١ & z \end{vmatrix} = ٢$$

【 بقية الأسئلة في الصفحة الثانية 】

II - Solid Geometry

Answer TWO ONLY of the Following Questions:

٤– (٧ Marks)

(a) **Complete each of the following to be a true statement:**

١) The angle between two skew lines is one of the angles between one of them and any other line passing through a point of this line ....

٢) If a line is perpendicular to a plane , then every plane containing this line is ....

٣) If two straight lines intersect a set of parallel planes, then the lengths of the line segments intercepted between these planes are ....

٤) If the length of the diagonal of a cube is $3\sqrt{3}$ cm , then the length of the diagonal of each face to this cube equals .... cm

(b) $XYZX'Y'Z'$ is an triangular inclined prism. If the point $L \in \overline{XX'}$ such that $\overline{XX'} \perp$ plane LYZ, prove that the face $YY'Z'Z$ is a rectangle. And if $m(\angle YY'X') = 30^\circ$ find the measure of the inclination angle of $\overline{XY}$ to the plane LYZ.

٥– (٧ Marks)

(a) Prove that " If the projection of a line inclined to a plane on the plane is perpendicular to a line in it, then the inclined line is perpendicular to the line in the plane ".

(b) X , Y are two parallel planes. If the points $A , B , H \in$ the plane X, $\overline{CD} \subset$ plane Y such that: $\overline{DH} // \overline{BC}$, $\overline{AD} \cap \overline{BC} = \{M\}$.

Prove that the points A , B , H are collinear.

And if M is the midpoint of $\overline{AD}$, AB = ٧ cm , find the length of $\overline{BH}$

٦– (٧ Marks)

M A B C is a triangular pyramid in which $\overline{MA}$, $\overline{MB}$, $\overline{MC}$ are mutually perpendicular and if $D \in \overline{BC}$ such that $\overline{AD} \perp \overline{BC}$ and $H \in \overline{AD}$ such that $\overline{MH} \perp \overline{AD}$

i) Prove that: $\dfrac{DB}{DH} = \dfrac{DA}{DC}$

ii) If MA = MB = MC determine the tangent of the plane angle of the dihedral angle between the two planes ABC and MBC

【 انتهت الأسئلة 】

الدور الأول - المرحلة الثانية	جمهورية مصر العربية وزارة التربية والتعليم امتحان شهادة إتمام الدراسة الثانوية العامة لعام ٢٠١٢ م ٤٠ ث.ع نموذج إجابة [الجبر والهندسة الفراغية بالإنجليزية	الدرجة العظمى (١٥)
عدد الصفحات (٦)		الدرجة الصغرى (---)

Total marks ٣٠/٢ = ١٥

The other solutions should be considered

I- Algebra

Answer to the first question (٨ Marks)	part(a) **four** marks, part(b) **four** marks

(A)

$$ {}^{n+2}P_r = 2 \times \frac{{}^{n+2}P_r}{r!} \quad \boxed{\text{one mark}} $$

$$ \Rightarrow \quad r! = 2! \quad \boxed{\text{half}} $$

$$ \therefore r = 2 \quad \boxed{\text{half}} $$

$$ \frac{{}^{n}C_3}{{}^{n}C_2} = \frac{5}{3} \qquad \Rightarrow \frac{n-3+1}{3} = \frac{5}{3} \quad \boxed{\text{one mark}} $$

$$ \Rightarrow n = 7 \quad \boxed{\text{half}} $$

$$ {}^{14}C_5 + {}^{10}P_1 = \frac{14 \times 13 \times 12 \times 11 \times 10}{5 \times 4 \times 3 \times 2 \times 1} + 10 $$

$$ = 2012 \quad \boxed{\text{half}} $$

(B)

$$ \text{The experssion} = \frac{1}{4 + 2\omega^2 + 3\omega} \ \boxed{\text{half}} + \frac{1}{5 + 3\omega + 4\omega^2} \ \boxed{\text{half}} $$

$$ = \frac{1}{(2\omega^2 + 2\omega + 2) + (2 + \omega)} \ \boxed{\text{half}} + \frac{1}{(3\omega^2 + 3\omega + 3) + (2 + \omega^2)} \ \boxed{\text{half}} $$

$$ = \frac{1}{2+\omega} + \frac{1}{2+\omega^2} $$

$$ = \frac{2 + \omega^2 + 2 + \omega}{(2+\omega)(2+\omega^2)} \ \boxed{\text{half}} $$

$$ = \frac{4 + (\omega^2 + \omega)}{4 + 2(\omega^2 + \omega) + 1} \ \boxed{\text{half}} $$

$$ = \frac{4-1}{4-2+1} \ \boxed{\text{half}} $$

$$ = 1 \ \boxed{\text{half}} $$

(تراعى الحلول الأخرى)

Answer to the **second** question (٨ marks)	part(A) **four** marks, part(B) **four** marks

(A)

$$z=\left(\frac{3}{2}+\frac{\sqrt{3}}{2}i\right)^6 \quad \boxed{\text{half}}$$

$$\therefore z=\left(\sqrt{3}\right)^6\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^6 \quad \boxed{\text{half}}$$

$$\therefore z=27\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^6 \quad \boxed{\text{half}}$$

$$\therefore z=27\left(\cos\pi+i\sin\pi\right) \quad \boxed{\text{half}}$$

Another solution to this part :

$$z=\left(1+2\cos^2\frac{\pi}{6}-1+2i\sin\frac{\pi}{6}\cos\frac{\pi}{6}\right)^6 \quad \boxed{\text{One mark}}$$

$$\therefore z=\left(2\cos\frac{\pi}{6}\right)^6\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^6 \quad \boxed{\text{half}}$$

$$\therefore z=27\left(\cos\pi+i\sin\pi\right) \quad \boxed{\text{half}}$$

$$\therefore z=27e^{\pi i}$$

$$\therefore \sqrt[3]{z}=3e^{\frac{(\pi+2\pi k)}{3}i} \quad \text{where } k=0,1,2 \quad \boxed{\text{one mark}}$$

When $k=0$ $\Rightarrow$ The first cubic root $=3e^{\frac{\pi}{3}i}$ $\boxed{\text{half}}$

When $k=1$ $\Rightarrow$ The second cubic root $=3e^{\pi i}$

When $k=2$ $\Rightarrow$ The third cubic root $=3e^{\frac{5\pi}{3}i}$ $\boxed{\text{half}}$

(B)

column (1) + column (2) + column (3)

$$\begin{vmatrix}2x+2 & 1 & 1\\ 2x+2 & 2x & 1\\ 2x+2 & 1 & 2x\end{vmatrix}=0 \quad \boxed{\text{one mark}} \quad \Rightarrow (2x+2)\begin{vmatrix}1 & 1 & 1\\ 1 & 2x & 1\\ 1 & 1 & 2x\end{vmatrix}=0 \quad \boxed{\text{half}}$$

column (3) - column (1)

$$2(x+1)\begin{vmatrix}1 & 1 & 0\\ 1 & 2x & 0\\ 1 & 1 & 2x-1\end{vmatrix}=0 \quad \boxed{\text{half}}$$

column (2) - column (1)

$$2(x+1)\begin{vmatrix}1 & 0 & 0\\ 1 & 2x-1 & 0\\ 1 & 0 & 2x-1\end{vmatrix}=0 \quad \boxed{\text{half}}$$

$\therefore 2(x+1)(2x-1)^2=0$ $\boxed{\text{half}}$ $\therefore x=-1$ $\boxed{\text{half}}$ or $x=\frac{1}{2}$ $\boxed{\text{half}}$

(تراعى الحلول الأخرى)

Answer to the **third** question (٨ marks)	part(a) **four** marks, part(b) **four** marks

(A) $T_{r+1} = {}^{3n}C_r \left(\frac{1}{2x}\right)^r (x^2)^{3n-r}$ **half**

$\Rightarrow T_{r+1} = {}^{3n}C_r \, 2^{-r} x^{6n-3r}$ **half**

$\therefore 6n-3r=0$ **half**

$\Rightarrow r=2n$ **half**

$\therefore$ The order of the term free of x is $(2n+1)$ and the term is T_{2n+1}

when $n=4$, the term free of x is T_9 and the middel term is T_7 **half**

$\frac{T_9}{T_7}=\frac{T_9}{T_8}\times\frac{T_8}{T_7}$ **half** $=\frac{12-8+1}{8}\times\frac{12-7+1}{7}\times\left(\frac{1}{2}\right)^2$ **half**

$=\frac{5}{8}\times\frac{6}{7}\times\frac{1}{4}=\frac{15}{112}$ **half**

(B) The equations are

$2x-y=1$ & $3y-2z=1$ & $-x+3z=2$ **half**

$$\Delta=\begin{vmatrix}2 & -1 & 0\\ 0 & 3 & -2\\ -1 & 0 & 3\end{vmatrix}=16$$ **half**

$$\Delta_x=\begin{vmatrix}1 & -1 & 0\\ 1 & 3 & -2\\ 2 & 0 & 3\end{vmatrix}=16$$ **half**

$$\Delta_y=\begin{vmatrix}2 & 1 & 0\\ 0 & 1 & -2\\ -1 & 2 & 3\end{vmatrix}=16$$ **half** ,

$$\Delta_z=\begin{vmatrix}2 & -1 & 1\\ 0 & 3 & 1\\ -1 & 0 & 2\end{vmatrix}=16$$ **half**

$x=\frac{\Delta_x}{\Delta}=\frac{16}{16}=1$ **half** , $y=\frac{\Delta_y}{\Delta}=\frac{16}{16}=1$ **half** , $z=\frac{\Delta_z}{\Delta}=\frac{16}{16}=1$ **half**

(تراعى الحلول الأخرى)

II- Solid Geometry

Answer to the **fourth** question (٧ marks)	part(A) **four** marks, part(B) **three** marks

A)(i) parallel to the other. **One mark**

(ii) perpendicular to that plane **One mark**

(iii) proportionl **One mark**

(iv) $3\sqrt{2}$ cm **One mark**

(B) $\because \overline{XX'} \perp$ the plane LYZ $\quad \therefore \overline{XX'} \perp \overline{YZ}$ **half**

$\because \overline{XX'} // \overline{YY'} \quad \therefore \overline{YY'} \perp \overline{YZ}$ **half** ......(1)

$\because$ The face YY′Z′Z is a parallelogramm, from the properties of the prism ....(2)

From (1) and (2) $\therefore$ The fac e YY′Z′Z is a rectangle **half**

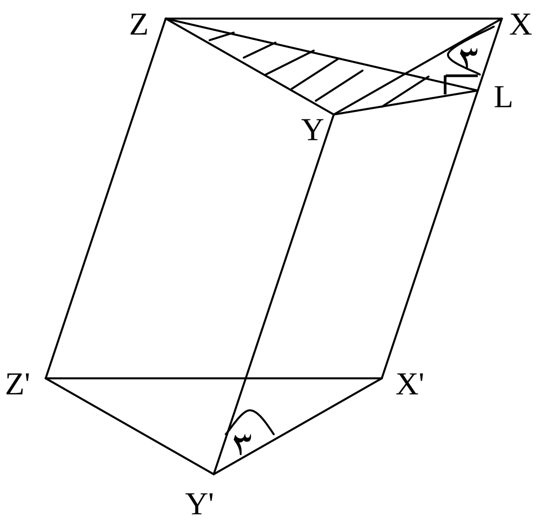
In the parallelogramm XX′Y′Y

$\because m(\angle YY'X') = 30^\circ \quad \Rightarrow m(\angle X'XY) = 30^\circ$ **half**

$\because \overline{XX'} \perp$ the plane LYZ $\quad \therefore \overline{LY}$ is the projection of $\overline{XY}$ on the plane LYZ

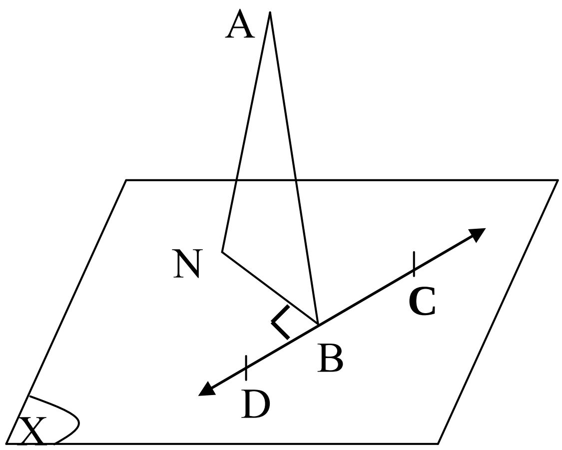
$\therefore \angle XYL$ is the inclination angle of $\overline{XY}$ on the plane LYZ **half**

$\therefore m(\angle XYL) = 60^\circ$ **half**



(تراعى الحلول الأخرى)

Answer to the **fifth** question (٧ marks)	part(A) **four** marks, part(B) **three** marks



(A) $\because \overleftrightarrow{AN} \perp$ the plane X

$\Rightarrow \overleftrightarrow{AN} \perp$ each straight line in the plane X

$\therefore \overleftrightarrow{AN} \perp \overleftrightarrow{CD}$ **one mark**

$\because \overleftrightarrow{NB} \perp \overleftrightarrow{CD}$

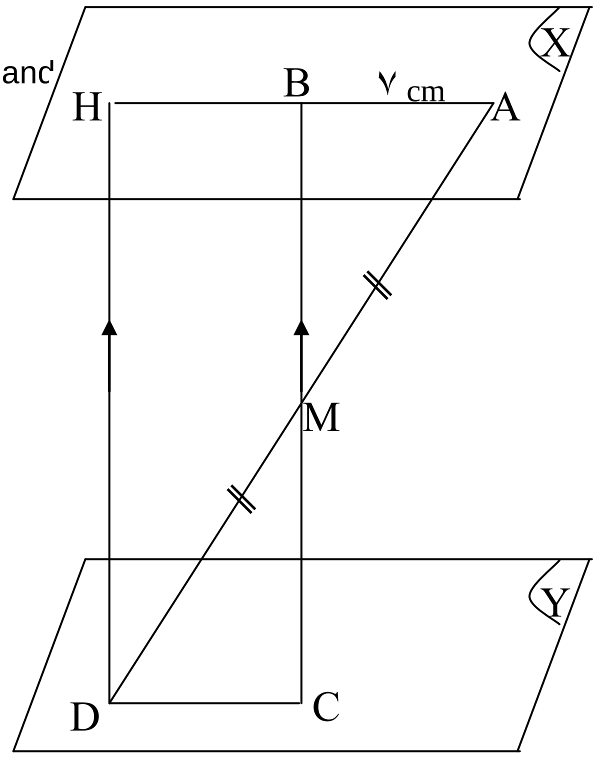
$\therefore \overleftrightarrow{CD} \perp$ each of $\overleftrightarrow{AN}$ and $\overleftrightarrow{NB}$ **one mark**

$\therefore \overleftrightarrow{CD} \perp$ the plane ANB **One mark**

$\therefore \overleftrightarrow{CD} \perp$ every line in the plane ANB

$\therefore \overleftrightarrow{CD} \perp \overleftrightarrow{AB}$ **One mark**

(B)



$\because \overline{AD} \cap \overline{BC} = \{M\}$

$\therefore$ The points A, B, C, D lie in the same plane, **half** which intersects the plane X at $\overleftrightarrow{AB}$ and the plane Y at $\overleftrightarrow{CD}$.

$\therefore \overleftrightarrow{AB} // \overleftrightarrow{CD}$ **half** (1)

$\because \overleftrightarrow{BC} // \overleftrightarrow{HD},\ \overleftrightarrow{BC} \cap \overleftrightarrow{HD} = \phi$

$\therefore \overleftrightarrow{BC}, \overleftrightarrow{HD}$ form the plane BCDH

This plane intersects the two parallel planes X and Y in $\overleftrightarrow{BH}$ and $\overleftrightarrow{CD}$, respectively.

$\therefore \overleftrightarrow{BH} // \overleftrightarrow{CD}$ **half** (2)

From (1), (2)

$\therefore \overleftrightarrow{AB} // \overleftrightarrow{BH}$ **half**

$\therefore$ A, B, H are collinear. **half**

$\because AM = MD,\ \overleftrightarrow{MB} // \overleftrightarrow{DH}$

$\therefore$ B is at the Middel of $\overline{AH}$

$\therefore BH = AB = 7$ cm **half**

(تراعى الحلول الأخرى)

Answer to the **sixth** question (٧ marks)

$\because \overline{MA} \perp$ each of $\overline{MB}, \overline{MC}$ $\therefore \overline{MA} \perp$ the plane MBC **half**

, $\overline{MD}$ is the projection of $\overline{AD}$ to the plane MBC **half**

, $\because \overline{AD} \perp \overline{BC}$

$\therefore \overline{MD} \perp \overline{BC}$ **half** $\therefore (MD)^2 = DB \times DC$ .... (1) **half**

$\because \overline{MH} \perp \overline{AD},\ \overline{AM} \perp \overline{MD}$ $\therefore (MD)^2 = DH \times DA$ .... (2) **half**

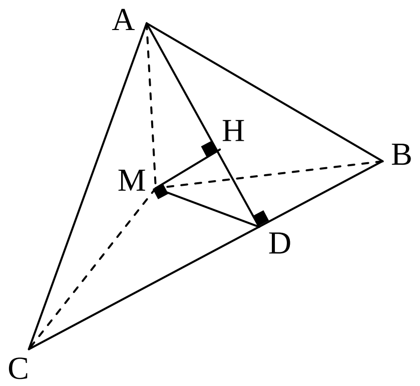
from (1) and (2) $\therefore DB \times DC = DH \times DA$ **half**

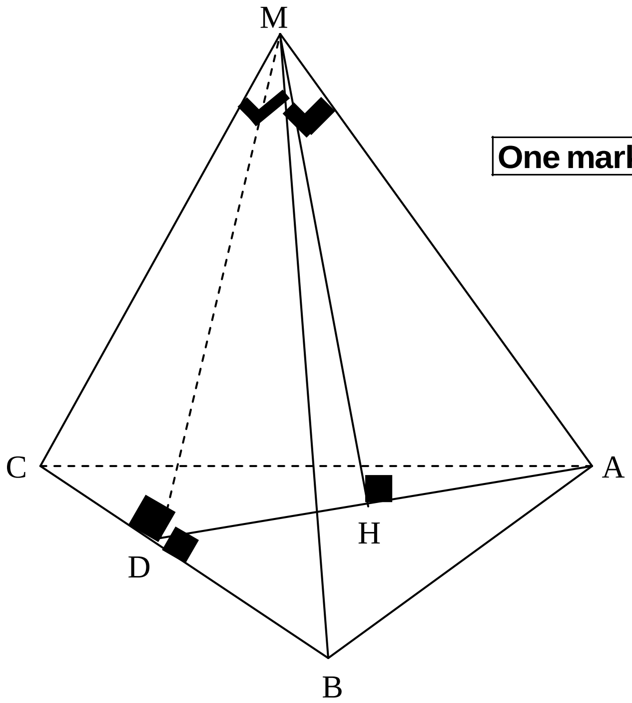
$\therefore \frac{DB}{DH} = \frac{DA}{DC}$ **half**

$\because \overline{AD} \perp \overline{BC},\ \overline{MD} \perp \overline{BC}$

$\therefore \angle ADM$ is the plane angle of the dihedral angle $(A - \overleftrightarrow{BC} - M)$ **One mark**

$\therefore \tan ADM = \frac{AM}{MD}$ **half** $= \frac{AM}{MB \sin 45^\circ}$ **half** $= \sqrt{2}$ **half**





(انتهى نموذج الإجابة)

(تراعى الحلول الأخرى)

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انتهى النموذج

ARAB REPUBLIC OF EGYPT
Ministry of Education
General Secondary Education Certificate Examination, 2011
[Second Stage – Second Session]
Algebra and Solid Geometry [Mathematics (2)] Time: 2 Hours

الجبر والهندسة الفراغية [رياضيات (٢)] باللغة الإنجليزية

[الأسئلة فى صفحتين]

Calculators are allowed

تنبيه مهم : يسلم الطالب ورقة امتحانية باللغة العربية مع الورقة المترجمة

I - Algebra

Note that: $1\ ,\ \omega\ ,\ \omega^2$ are the cubic roots of unity, and $i^2 = -1$

Answer ONLY TWO of the Following Questions:

1 - a. If $^{14}C_r = {}^{14}C_{3r+2}$, $^nP_r = 720$. **Find** the value of : $\underline{|n-2r}$

b. Find in simplest form the value of :

$$\left(1 - \frac{3}{\omega^2} + \omega^2\right)\left(1 + \omega^4 - \frac{2}{\omega}\right)$$

2 - a. Solve by using Cramer's rule the following system of equations :

$$2x + y + 3z = -2\ ,\quad 3x - 2y - 2z = 7\ ,\quad x - y + z = 1$$

b. In the expansion of $\left(2x + \frac{3}{x^2}\right)^n$ according to descending of the power of x, If the ninth and the tenth terms are equal, and the ratio between the sixth term and the seventh term is 8 : 15 **Find** the value of n, and **prove that** there is no term free of x in this expansion

3 - a. Put the number $Z = \frac{8}{1+\sqrt{3}\,i}$ in the trigonometry form , and hence **find** its two square roots in the exponential form .

b. Without expanding **find** the value of the following determinant:

$$\begin{vmatrix} 1 & x & y \\ x & 1+x^2 & xy \\ y & xy & 1+y^2 \end{vmatrix}$$

[بقية الأسئلة فى الصفحة الثانية]

II - Solid Geometry

Answer ONLY TWO of the Following Questions:

4- a. Complete each of the following to be a true statement:

1) If a line is parallel to each of two intersecting planes, then……
2) The two lines perpendicular to the same plane are ………… .
3) If a line contained in one of two perpendicular planes is perpendicular to their lines of intersection, then this line is .....
4) If the surface area of a cube is 150 cm^2 , then the length of its diagonal is ……….. .

b. DABC is a triangular pyramid in which $\overline{DA}$ is drawn perpendicular to both $\overline{AB}$, $\overline{AC}$ also $\overline{AE}$ is drawn perpendicular to $\overline{BC}$ where $E \in BC$ **Prove that** $\overline{BC} \perp$ plane DAE.

5- a. Prove that " If a line inclined to a plane is perpendicular to a line in the plane, then its projection on the plane is perpendicular to the line in the plane .

b. A , B , C , D are four points non-coplanar . Plane X is drawn intersects $\overline{AB}$, $\overline{AC}$, $\overline{AD}$ at the points x , y , z respectively such that : $\frac{Ax}{AB} = \frac{Ay}{AC} = \frac{Az}{AD} = \frac{1}{4}$ **Prove that** the plane X parallel to plane BCD and if BC = 12 cm , CD = 16 cm and BD = 20 cm , **find** the surface area of $\Delta\ x\,y\,z$

6- MABCD is a right quadrilateral pyramid whose base ABCD where AB = $6\sqrt{2}$ cm , and the length of its lateral edge =12 cm , **find**:

1) The height of the pyramid.
2) The measure of the angle of inclination of $\overline{MB}$ to the plane of the base ABCD.
3) The tangent of the angle between the two planes MAB,ABCD

[انتهت الأسئلة]

الدور الأول

جمهورية مصر العربية
وزارة التربية والتعليم
امتحان شهادة إتمام الدراسة الثانوية العامة لعام ٢٠١١ م

المرحلة الثانية

نموذج إجابة امتحان مادة [الجبروالهندسة الفراغية (٢) بالإنجليزية]

الدرجة الكلية : (٣٠) درجة ثم تقسم على (٢) لتصبح الدرجة الفعلية (١٥) درجة

إجابة السؤال الأول (ثمانى درجات)	**الفقرة (a) أربع درجات ، الفقرة (b) أربع درجات**

(١)

(a)

${}^{x+y}P_{٤} = {}^{٦}P_{٤}$ ٠.٥ $\therefore$ $x + y = ٦$ ………….. (١) ٠.٥

$\lfloor\underline{2x+y} = \lfloor\underline{7}$ ٠.٥ $\therefore$ $٢x + y = ٧$ ………….. (٢) ٠.٥

From (١) , (٢) $x = ١$ ٠.٥ & $y = ٥$ ٠.٥

${}^{٥}C_{٢} = \dfrac{5\times4}{2\times1}$ ٠.٥ $= ١٠$ ٠.٥

(b) The expression $= \left(\dfrac{1}{1+3\omega^2} - \dfrac{1}{1+3\omega}\right)^2$ ١

$$\left(\frac{1+3\omega-1-3\omega^2}{(1+3\omega^2)(1+3\omega)}\right)^2$$ ٠.٥

$$= \left(\frac{3(\omega-\omega^2)}{1+3\omega+3\omega^2+9\omega^3}\right)^2$$ ٠.٥

$$= \left[\frac{3\times\pm\sqrt{3}\,i}{1+3(\omega+\omega^2)+9}\right]^2$$ ١

$$= \frac{27\,i^2}{(10-3)^2}$$ ٠.٥ $$= \frac{-27}{49}$$ ٠.٥

تراعى الحلول الأخرى

إجابة السؤال الثاني (ثمانى درجات)	الفقرة (a) أربع درجات ، الفقرة (b) أربع درجات

(٢)

(a) $\Delta = \begin{vmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & -3 & 1 \end{vmatrix} = ٨$ 　 ١

$\Delta_x = \begin{vmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & -3 & 1 \end{vmatrix} = ٨$ 　 ٠.٥ 　　 $\therefore x = \dfrac{\Delta_x}{\Delta} = \dfrac{8}{8} = 1$ 　 ٠.٥

$\Delta_y = \begin{vmatrix} 3 & 3 & 1 \\ 1 & 2 & 1 \\ 1 & 6 & 1 \end{vmatrix} = -٨$ 　 ٠.٥ 　　 $\therefore y = \dfrac{\Delta_y}{\Delta} = \dfrac{-8}{8} = -1$ 　 ٠.٥

$\Delta_z = \begin{vmatrix} 3 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & -3 & 6 \end{vmatrix} = ١٦$ 　 ٠.٥ 　　 $\therefore z = \dfrac{\Delta_z}{\Delta} = \dfrac{16}{8} = 2$ 　 ٠.٥

(b) $\dfrac{T_3}{T_2} = \dfrac{720}{240}$ 　 ٠.٥ 　　 $\dfrac{n-2+1}{2} \times \dfrac{y}{x} = 3$ 　 ٠.٥

$(n - ١) \times \dfrac{y}{x} = ٦$ .... (١)

$\dfrac{T_4}{T_3} = \dfrac{1080}{720}$ 　 ٠.٥ 　　 $\dfrac{n-3+1}{3} \times \dfrac{y}{x} = \dfrac{3}{2}$ 　 ٠.٥

$(n - ٢) \times \dfrac{y}{x} = \dfrac{9}{2}$ .... (٢)

From (١) , (٢) by dividing : $\dfrac{n-1}{n-2} = \dfrac{6 \times 2}{9}$ 　 $\therefore$ n = ٥ 　 ٠.٥

From (١) : $y = \dfrac{3}{2}x$ (٣) 　 ٠.٥ 　 $\because T_{٢} = ٢٤٠$ 　 $\therefore {}^{n}C_{١}\, y\, x^{n-١} = ٢٤٠$

${}^{٥}C_{١} (\dfrac{3}{2} x) x^{٤} = ٢٤٠$ 　 $\therefore x^{٥} = ٣٢$ 　 x = ٢ 　 ٠.٥ 　 and from (٣) , y = ٣ 　 ٠.٥

تراعى الحلول الأخرى

إجابة السؤال الثالث (ثمانى درجات)	الفقرة (a) أربع درجات ، الفقرة (b) أربع درجات

(٣)

(a)

$$Z = \frac{8}{1-\sqrt{3}\,i} \times \frac{1+\sqrt{3}\,i}{1+\sqrt{3}\,i}$$ ٠.٥ $$= 2 + 2\sqrt{3}\,i$$ ٠.٥

x = ٢ , y = ٢ $\sqrt{3}$ $\therefore$ r = ٤ ٠.٥

$$\cos\theta = \frac{1}{2} \;,\; \sin\theta = \frac{\sqrt{3}}{2} \quad \therefore \theta = \frac{\pi}{3}$$ ٠.٥

z = ٤ $\left(\cos\frac{\pi}{3} + i\ \sin\frac{\pi}{3}\right)$ ٠.٥ z = ٤ $e^{\frac{\pi}{3}i}$ ٠.٥

$$\therefore \sqrt{Z} = 2\,e^{\left(\frac{\frac{\pi}{3}+2\pi r}{2}\,i\right)}$$ ٠.٥ where r = ٠ , ١

At r = ٠ $\therefore$ the first root of $z = 2\,e^{\frac{\pi}{6}i}$ ٠.٥

At r = ١ $\therefore$ the second root of $z = 2\,e^{\frac{7\pi}{6}i}$ ٠.٥

(b)

R ١ - R٣ $\therefore$ $\Delta = \begin{vmatrix} y & x-z & -y \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$ ١

R ١ + R ٢

$\Delta = \begin{vmatrix} 2y & 2x & 0 \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$ ١ = ٢ $\begin{vmatrix} y & x & 0 \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$ ١

R ٢ - R١ : Δ = ٢ $\begin{vmatrix} y & x & 0 \\ 0 & z & y \\ z & z & x+y \end{vmatrix}$ ٠.٥

R ٣ - R٢ : Δ = ٢ $\begin{vmatrix} y & x & 0 \\ 0 & z & y \\ z & 0 & x \end{vmatrix}$ ٠.٥

تراعى الحلول الأخرى

ثانياً : الهندسة الفراغية

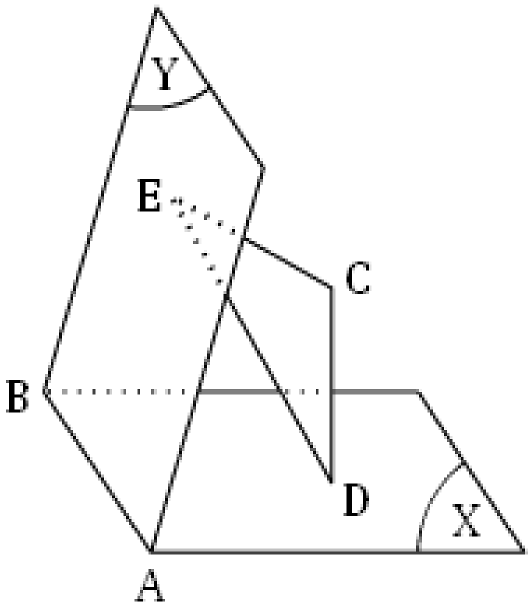
إجابة السؤال الرابع (سبع درجات)	الفقرة (a) أربع درجات ، الفقرة (b) ثلاث درجات

(٤)

(a)

No. of part	The correct answer	The mark of each item
١	parallel	One mark
٢	Perpendicular to the line	One mark
٣	Perpendicular to its plane	One mark
٤	$5\sqrt{3}$ cm	One mark

(b)



$\because \overleftrightarrow{CD} \perp$ plane x

$\therefore \overleftrightarrow{CD} \perp \overleftrightarrow{AB}$ ١

$\overleftrightarrow{CE} \perp$ plane y

$\therefore \overleftrightarrow{CE} \perp \overleftrightarrow{AB}$ ١

$\therefore \overleftrightarrow{AB} \perp$ to both $\overleftrightarrow{CD}$, $\overleftrightarrow{CE}$ ٠.٥

$\therefore \overleftrightarrow{AB} \perp$ the plane CDE ٠.٥

$\therefore \overleftrightarrow{AB} \perp \overleftrightarrow{DE}$

تراعى الحلول الأخرى

إجابة السؤال الخامس (سبع درجات)	الفقرة (a) ثلاث درجات ، الفقرة (b) أربع درجات

(٥)

(a)

Theorem's Proof (٣ marks)

(b)

$\because \overleftrightarrow{AC}$ // plane X

$\overleftrightarrow{AC} \subset$ the plane ABC and intersect plane X at $\overleftrightarrow{EF}$ ٠.٥

$\therefore \overleftrightarrow{EF} // \overleftrightarrow{AC}$ ٠.٥

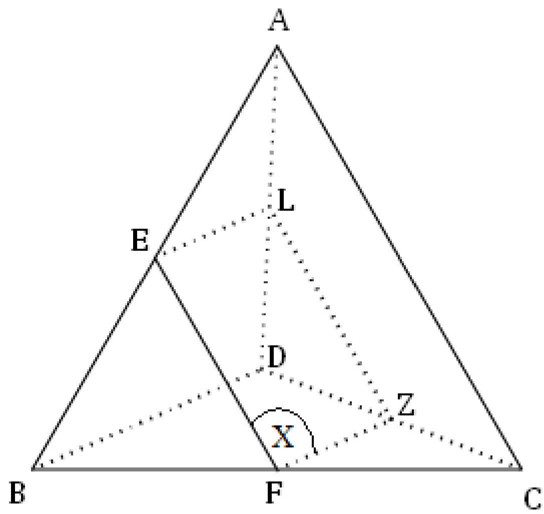
$\therefore \frac{BE}{BA} = \frac{BF}{BC} = \frac{EF}{AC}$ ....(١) ٠.٥

$\because \overleftrightarrow{BD}$ // plane X

$\overleftrightarrow{BD} \subset$ the plane ABD , and intersect plane X at $\overleftrightarrow{EL}$ ٠.٥

$\therefore \overleftrightarrow{EL} // \overleftrightarrow{BD}$ ٠.٥

$\therefore \frac{AE}{AB} = \frac{AL}{AD} = \frac{EL}{BD}$ .... (٢) ٠.٥



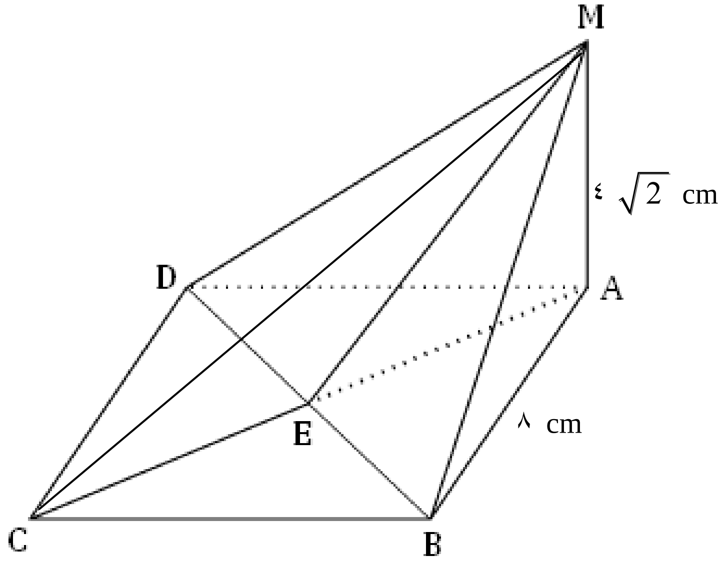
From (١) , (٢)

$\frac{EF}{AC} + \frac{EL}{BD} = \frac{BE}{BA} + \frac{AE}{AB}$ ٠.٥

$= \frac{BE + AE}{AB} = \frac{AB}{AB}$ ٠.٥ $= ١$

تراعى الحلول الأخرى

إجابة السؤال السادس (سبع درجات)



I) Join $\overline{ME}$ ٠,٥

$\because$ $\overline{ME}$ is inclined to the plane ABCD

$\because$ its projection $\overline{AE} \perp \overleftrightarrow{BD}$ ٠,٥

$\therefore$ $\overline{ME} \perp \overleftrightarrow{BD}$ ٠,٥

$\therefore$ $m(\angle A\text{-}\overleftrightarrow{BD}\text{-}M) = m(\angle AEM)$ ٠,٥

$\because$ AC = ٨ $\sqrt{2}$ cm

$\therefore$ AE = ٤ $\sqrt{2}$ cm ٠,٥

$\therefore$ $\tan(\angle AEM) = \dfrac{4\sqrt{2}}{4\sqrt{2}} =$ ١ ٠,٥

$\therefore$ $m(\angle AEM) =$ ٤٥° ٠,٥

$\therefore$ $m(\angle A\text{-}\overleftrightarrow{BD}\text{-}M) =$ ٤٥° ٠,٥

II) tan $(\angle MCA) = \dfrac{MA}{AC}$ ٠,٥ $= \dfrac{4\sqrt{2}}{8\sqrt{2}}$ ٠,٥ $= \dfrac{1}{2}$ ٠,٥

III) $\because$ $\overline{MA} \perp$ plane (ABCD)

$\therefore$ $\overline{MA} \perp \overleftrightarrow{BD}$ ٠,٥

$\because$ $\overline{AC} \perp \overleftrightarrow{BD}$

$\therefore$ $\overleftrightarrow{BD} \perp$ plane (MAC) ٠,٥

$\therefore$ $\overleftrightarrow{BD} \subset$ plane MBD ٠,٥

$\therefore$ Plane MBD $\perp$ plane MAC

تراعى الحلول الأخرى

[انتـهـت الإجابة]

ARAB REPUBLIC OF EGYPT
Ministry of Education
General Secondary Education Certificate Examination, ٢٠١٢
[Second Stage – Second Session]

Algebra and Solid Geometry 【 Mathematics (٢) 】 Time: ٢ Hours

الجبر والهندسة الفراغية [رياضيات (٢)] باللغة الإنجليزية

تنبيه مهم : ١- يسلم الطالب ورقة امتحانية باللغة العربية مع الورقة المترجمة . 【 الأسئلة في صفحتين 】
٢- الإجابات المكررة عن أسئلة الاختيار من متعدد والصواب والخطأ لن تقدر ويتم تقدير الإجابة الأولى فقط .

Calculators are allowed: الدرجة الفعلية = مجموع الدرجات ÷ ٢

Note that: 1 , ω , ω^2 are the cubic roots of unity, and: $i^2 = -1$

I – Algebra

Answer TWO ONLY of the Following Questions:

١ – (٨ Marks)

(a) If ${}^{15}C_{3r} = {}^{15}C_{r+3}$, ${}^{(n+2)}C_8 : {}^{(n-2)}P_4 = 3:40$,

find the value of ${}^{n}C_{r+1} : {}^{n}C_{r}$

(b) Find the value of the expression: $\left(\dfrac{1+i}{1+2\omega}\right)^4 + \left(\dfrac{1-i}{1+2\omega^2}\right)^4$

٢ – (٨ Marks)

(a) If $z_1 = \left(\sin\dfrac{\pi}{9} + i\cos\dfrac{\pi}{9}\right)^5$, $z_2 = \left(\cos\dfrac{\pi}{9} + i\sin\dfrac{\pi}{9}\right)^4$

and the complex number $z = \dfrac{z_1}{z_2}$, find the two square roots of the number z in the exponential form.

(b) Using Cramer's Rule, solve the following equations:

$$x - y + 2z = -1 \ , \quad x + y = 1 \ , \quad y - 2z = 2$$

٣ – (٨ Marks)

(a) Find the order and the value of the term free of x in the expansion of $(x^3 - \frac{1}{x})^8$, according to the descending power of x, then find the ratio between the middle term and its following term at $x = ٢$.

(b) Without expanding the determinant, prove that:

$$\begin{vmatrix} y & x & x+y \\ y+3 & 3 & y \\ 3 & x+3 & x \end{vmatrix} = 12xy$$

[بقية الأسئلة في الصفحة الثانية]

II - Solid Geometry

Answer TWO ONLY of the Following Questions:

٤– (٧ Marks)

(a) **Complete each of the following to be a true statement:**

١) If a line not belonging to a plane is parallel to a line in the plane, then it is ....

٢) If a line is perpendicular to each of two coplanar non-parallel lines, then it is ....

٣) If a line inclined to a plane is perpendicular to a line in the plane, then its projection on the plane is ....

٤) If the altitude length in a regular triangular pyramid is $\sqrt{6}$ cm, then the sum of its edges length equals .... cm.

(b) CAB and DAB are two right-angled triangles at A lying in two different planes. If X, Y, N, M are the mid-points of $\overline{CA}$, $\overline{CB}$, $\overline{DB}$, $\overline{DA}$, respectively, prove that:

i) $\overleftrightarrow{MN}$ // the plane CAB. ii) The shape XYNM is a rectangle.

٥– (٧ Marks)

(a) Prove that: "If a line is perpendicular to a plane, then every plane containing this line is perpendicular to this plane".

(b) ABC is a triangle in which AC = ١٠ cm, $m(\angle BAC) = 60^\circ$. We draw $\overline{CD}$ perpendicular to the plane ABC such that CD = ٥ cm. $\overline{DH}$ is drawn perpendicular to $\overline{AB}$ such that $H \in \overline{AB}$. Calculate the length of $\overline{DH}$ and find the measure of the angle of inclination of $\overline{DH}$ to the plane ABC.

٦– (٧ Marks)

MABCD is a right quadrilateral pyramid whose vertex is (M), the side length of its base is 13 cm. The Point (N) is the center of its base and the point (O) is the midpoint of $\overline{AB}$. If $m(\angle M - \overleftrightarrow{AB} - D) = \theta^\circ$ where $\sin\theta = \dfrac{12}{13}$, find:

i) The length of the altitude of the pyramid.

ii) The lateral area of the pyramid.

iii) The measure of the plane angle of the dihedral angle between the two planes OMN , BMN.

【 انتهت الأسئلة 】

ARAB REPUBLIC OF EGYPT [٧٥] ش.ع / أول / ج

Ministry of Education

General Secondary Education Certificate Examination, 2011

[Second Stage – First Session]

Algebra and Solid Geometry [Mathematics (2)] Time: 2 Hours

الجبر والهندسة الفراغية [رياضيات (٢)] باللغة الإنجليزية

Remark: Calculators are permitted.

تنبيه هام : يسلم الطالب ورقة امتحانية باللغة العربية مع الورقة المترجمة [الأسئلة في صفحتين]

I - Algebra

Note that: 1 , ω , ω^2 are the cubic roots of unity, and $i^2 = -1$

Answer TWO ONLY of the Following Questions:

1- a) If ${}^{x+y}P_4 = 360$, $\underline{|2x + y} = 5040$

Find the value of : ${}^{y}C_{2x}$

b) Find the numerical value of the expression:

$$\left(\frac{1}{\omega^2(\omega+3)} - \frac{1}{1+3\omega^4}\right)^2$$

2- a) Using Cramer's method to solve the following equations:

$3x + 2y + z = 3$, $x + y + z = 2$, $x - 3y + z = 6$

b) In the expansion of $(x+y)^n$ in descending power of x , if T_2 , T_3 , T_4 are respectively 240 , 720 , 1080

Evaluate the value of each of x , y , n ?

3 - a) Put the number $z = \dfrac{8}{1-\sqrt{3}\,i}$ in the trigonometry form and hence **find** its two square roots in the exponential form

b) Without expanding the determinant, **prove that:**

$$\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} = 2 \begin{vmatrix} y & x & 0 \\ 0 & z & y \\ z & 0 & x \end{vmatrix}$$

[بقية الأسئلة في الصفحة الثانية]

II - Solid Geometry

Answer Two ONLY of the Following Questions:

4- a) Complete each of the following to be a true statement:

1) If a plane intersects two parallel planes, then its lines of intersection with these two planes are ....
2) If a line inclined to a plane and its projection on the plane is perpendicular to a line in the plane, then this inclined line is ....
3) The line perpendicular to each of two intersecting lines at this point of intersection is ....
4) If the volume of a cube is $125\ cm^3$, then the length of its diagonal equals ....

b) X and Y are two intersecting planes at $\overleftrightarrow{AB}$, from a point C that does not belong to any of them , drawn $\overrightarrow{CD}$ perpendicular to the plane X and intersect it at D , and drawn $\overrightarrow{CE}$ perpendicular to the plane Y and intersect it at E

Prove that: $\overleftrightarrow{AB} \perp DE$

5- a) Prove that " If a line is parallel to a plane, then its parallel to all lines of intersection of this plane with the planes containing the given line "

b) ABCD is a triangular pyramid, the plane X is drawn parallel to $\overleftrightarrow{AC}$, $\overleftrightarrow{BD}$ intersects $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$ in E , F , Z , L respectively. **Prove that:** $\dfrac{EF}{AC} + \dfrac{EL}{BD} = 1$

6- ABCD is a square of side length 8 cm, if its diagonals intersect at E, AM is drawn perpendicular to the plane of the square such that $AM = 4\sqrt{2}$ cm.

1) **Find** m ($\angle$ A – $\overleftrightarrow{BD}$ – M)

2) **Find** The tangent of the angle of inclination of $\overline{MC}$ to the plane ABCD

3) **Prove that :** the plane MAC $\perp$ the plane MBD

[انتهت الأسئلة]

ARAB REPUBLIC OF EGYPT

Ministry of Education

General Secondary Education Certificate Examination, 2010

[Second Stage – First Session]

Mathematics (2) Algebra and Solid Geometry **Time :2 hours**

الجبر والهندسة الفراغية [رياضيات (٢)] باللغة الإنجليزية

Remark **Calculators are allowed.**

تنبيه هام : يسلم للطالب ورقة امتحانية باللغة العربية مع الورقة المترجمة (الأسئلة في صفحتين)

I - Algebra

Note that: 1 , ω , ω^2 are the cubic roots of unity, and: $i^2 = -1$

Answer TWO ONLY of the Following Questions :

1 - a) Find the value of : $\left(1 - \frac{5}{\lfloor^2} + \lfloor^2\right)\left(1 + \lfloor - \frac{3}{\lfloor}\right)$

b) In the expansion of $\left(x^2 + \frac{1}{8x}\right)^{13}$, according to the descending order of the powers of x :

i) **prove that** there is no term including x .

ii) If the fourth and eleventh terms are equal , then **find** the value of x.

2 - a) Solve the following equations Using Cramer's method :

$$x + 3y = 8 \ , \quad 3y - 2z = 6 \ , \quad x + 3z = 2$$

b) If X is non empty set whose number of its elements is n , and if :

$Z_1 = \{ \{ a, b, c \} : a, b, c \in X \}$, $Z_2 = \{ (a, b) : a, b \in X, \ a \neq b \}$

and the number of elements of Z_1 = the number of elements of Z_2 , then **find** the value of n. And if

$$4k\,\underline{|2k-1} = \frac{32}{9} \times \frac{{}^{11}c_3 + {}^{11}c_4}{{}^{12}c_3} + 5n$$, **find** the value of k .

3 - a) If $Z_1 = \sqrt{2}\left(\cos \frac{b}{4} - i \sin \frac{b}{4}\right)$, $Z_2 = 1 + i$,

find the exponential form of the number Z where $Z = \frac{Z_1^2}{Z_2}$

b) **Put** M in a triangular form , hence **find** its value given that :

$$M = \begin{vmatrix} 0 & 4 & 4 \\ 3 & 2 & 4 \\ 1 & 8 & 11 \end{vmatrix} + \begin{vmatrix} 0 & 4 & 4 \\ -3 & 5 & 0 \\ 1 & 4 & 7 \end{vmatrix} + 4\begin{vmatrix} 1 & 1 & 1 \\ 1 & 7 & 4 \\ 0 & 4 & 7 \end{vmatrix}$$

[بقية الأسئلة في الصفحة الثانية]

II - Solid Geometry

Answer Two ONLY of the Following Questions:

4- a) **Complete each of the following to be a true statement :**

i) The two lines which are each parallel to a third line in space are ....

ii) The two bases of a prism are parallel and ....

iii) The angle between a line segment and a plane is the angle between the line segment and ....

iv) If the sum of edges lengths of a regular triangular pyramid equals 18 cm , then its total surface area equals ....

b) M A B C is a triangular pyramid. Its base is the triangle A B C. Draw a plane which is parallel to the base of the pyramid and intersects $\overline{MA}$ at D, $\overline{MB}$ at H and $\overline{MC}$ at O , if the perimeter of the triangle A B C = 3 times the perimeter of the triangle D H O , then find M D : D A

5- a) **Prove that** " if a line inclined to a plane such that its projection in the plane is perpendicular to a line in it , then the inclined line is perpendicular to this line in the plane " .

b) X Y Z is a triangle in which X Y = X Z = 10 cm , Y Z = 12 cm . Let M be mid-point of $\overline{YZ}$, then draw $\overline{XN}$ perpendicular to the plane of the triangle such that X N = 8 cm.

i) **Prove that** : $\overline{NM} \perp \overline{YZ}$

ii) **Find** m($\angle X - \overleftrightarrow{YZ} - N$)

6- A B C D is a rectangle , its diagonals intersect at M , $\overline{MH}$ ‘ the plane of the rectangle such that $MH = \frac{1}{2} BC$, X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{CD}$

i) **Find** the measure of the dihedral angle between the two planes HAB and ABCD .

ii) **Find** the line of intersection of the two planes HAB and HCD , give the reasons

iii) **Prove that** the two planes HAB and HCD are perpendicular.

=.=.=.=.=.=.=

[انتهت الأسئلة]

ARAB REPUBLIC OF EGYPT ٢٧ ث.ع / ثان
Ministry of Education
General Secondary Education Certificate Examination,2010
Second Stage - second Session

Mathematics (2) Algebra and Solid Geometry **Time : 2 hours**

الجبر والهندسة الفراغية [رياضيات (٢)] باللغة الإنجليزية

تنبيه هام : يسلم للطالب ورقة امتحانية باللغة العربية مع الورقة المترجمة (**الأسئلة فى صفحتين**)

Calculators are Permitted

I – Algebra

Note that: 1 , ω , ω^2 are the cubic roots of unity, and: $i^2 = -1$

Answer Two ONLY of the Following Questions:

1- a) Solve the following equations using Cramer's method.

$x - y = -2$, $2y - 3z = -1$, $x + z = 0$

b) In the expansion of $(x^3 + \frac{1}{x})^{17}$, according to the descending order of the powers of x :

i) Prove that there is no term free of x.

ii) Find the value of x which makes the two middle terms are equal .

2- a) Find the values of :

$$(2i^4 - \frac{2i^2}{\omega^2} + 3\omega^2)^2 + (\frac{5}{\omega^3} + 6\omega^4 + 5\omega^2)^2$$

b) If $4 \times {}^{n+1}C_{r+1} = 9 \times {}^{n}C_{r}$ and $\lfloor n-3 = 120$, **find** the values of ${}^{n}P_{r} : {}^{n}C_{r}$

3- a) Put the numbers $Z_1 = (\cos\frac{4\pi}{3} - i\sin\frac{4\pi}{3})$, $Z_2 = \sqrt{3} - i$

In trigonometric form , then find the modulus and the principal amplitude of the number Z where $Z = Z_1 Z_2$

b) Without expanding the determinate **, find** the value of :

$$\begin{vmatrix} 3 & 5 & 6 \\ 8 & -4 & 12 \\ 4 & 7 & 5 \end{vmatrix} - \begin{vmatrix} -2 & 1 & -3 \\ 12 & 20 & 24 \\ 4 & 7 & 5 \end{vmatrix}$$

[بقية الأسئلة فى الصفحة الثانية]

Answer Two ONLY of the Following Questions:

4- a) Complete each of the following to be a true statement .

i) If a line contained in one of two perpendicular planes is perpendicular to their line of intersection then this line is …………….

ii) The vertical straight lines are all …………………

iii) If two straight lines L_1 , L_2 are skew lines then $L_1 \cap L_2$ =…………………

iv) The least number of planes which determine a solid is ……………

b) In the rectangular parallelepiped , if the surface area of three faces which intersect at the same point equals 2 , 3 , 6 cm^2 . Find the square of the length of the diagonal of the rectangular parallelepiped .

5- a) Prove that " if a line is parallel to a plane , then it is parallel to every line of the intersection of this plane with the planes containing the given line .

b) X , Y , Z are three parallel planes , the straight line L intersects them at A , B , C respectively and the straight line M intersects them at D , E , O respectively , and L , M lie on the same plane such that 4 DE = 3 EO , AD = 4 cm , CO = 11 cm .

Calculate the length of $\overline{BE}$.

6- MABCD is a right quadrangular pyramid with square base ABCD , $\overline{AC} \cap \overline{BD} = \{N\}$, the side length of square base = the length of lateral edge of the pyramid = $8\sqrt{2}$ cm .

Find :

i) The length of the altitude of the pyramid MABCD .

ii) The measure of the angle of inclination of $\overline{MA}$ with the base ABCD

iii) The measure of the dihedral angle between the two planes MAB and ABCD

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[انتـهـت الأسـئـلـة]

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[انتـهـت الأسـئـلـة]